MULTIPLE TARGET DISCRIMINATION
USING E-PULSE TECHNIQUES

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I. Introduction

Radar target discrimination based on the E-pulse technique [1] requires knowledge of the complex natural mode frequencies of the targets of interest. In practice, these frequencies are extracted from time domain measurements of a target's late time response. The measurements are made in a laboratory environment to achieve a signal-to-noise ratio sufficient for mode extraction. The E-pulse for each target is synthesized from the measured natural frequencies [2]. Discrimination is accomplished by successively convolving the E-pulses with the late time response of the unknown target. The correct E-pulse will yield a convolved response that has smaller energy than other incorrect E-pulses in the E-pulse library. This scheme has been demonstrated in the laboratory when only a single target is illuminated with the transient waveform.

The case of two or more targets is more problematic. When two targets are illuminated together they act as a system with natural frequencies different from those of the isolated targets. Mutual coupling could thus reduce the effectiveness of discrimination since the E-pulses are generated from measurement of isolated targets. To study this effect, the special case of a system of parallel thin cylinders was investigated both theoretically and experimentally.

II. Analysis

By enforcing the boundary condition on the tangential electric field at the surface of each of N perfectly conducting scatterers in free space, one arrives at the system of integral equations

$$1. \qquad \hat{t}_m \cdot \sum_{n=1}^{N} \int_{S_n} \bar{G}(\vec{r},\vec{r}',s) \cdot \vec{K}_n(\vec{r}',s)\, dS' = -\hat{t}_m \cdot \vec{E}^i(\vec{r},s)$$

where $\hat{t}$ is a unit vector tangent to the surface of the m^{th} scatterer, $\bar{G}$ is the free space electric dyadic Green's function, $\vec{K}_n$ is the surface current on the n^{th} scatterer, $\vec{E}^i$ is the impressed electric field, $\vec{r}$ and $\vec{r}'$ are the field point and source point, and s is the complex frequency. The natural mode frequencies s_n and currents of the system are defined when $\vec{E}^i = 0$. The response of the system in the late time can be represented as natural mode series of exponentials with frequencies s_n.

The start of late time for the system of scatterers is the maximum time required for the radiation from the part of the scatterer last excited by the impressed field to travel to all other parts of the system and then to the observation point. This is when the natural mode series is valid. However, prior to the beginning of the system late time, there are portions of the scattered signal that are composed of the natural frequencies of the isolated targets. Thus, for targets separated sufficiently in range it is possible to discriminate using a library of E-pulses for isolated targets and a time gating scheme. If several targets are at the same range but separated sufficiently in cross range, one can time gate so as to exclude the system late time and perform successive convolutions of E-pulses to ascertain the system composition. The E-pulse discrimination technique is impractical if E-pulses based on system *sn*'s must be included in the library since the system can have innumerable configurations. For cases where the targets are too close to allow time gating, only the system *sn*'s are available. One must now determine if the system *sn*'s are sufficiently different from the *sn*'s of the isolated targets to prevent discrimination.

To address this question, a system of two thin parallel cylinders was analyzed as a special case of a multiple target system. To solve for the natural mode frequencies, equation (1) was rewritten in Hallen's form and via the moment-method a matrix equation with s as a parameter was generated. The *sn*'s were found by numerically searching for values of s that made the determinant of the matrix zero. The natural mode currents were found by computing the matrix null space.

The natural mode current on each cylinder in a system of cylinders was found to be nearly the same as the natural mode current of the isolated cylinder. By using the current distribution of the isolated cylinder a transcendental equation in s was obtained. This was advantageous since the current distribution was fixed and permitted only a few solutions. This simplified the numerical search.

A perturbation approach for nearly degenerate coupling was also used for the case of identical cylinders [3]. Here, a sinusoidal current was assumed, and the Green's function was approximated as the first two terms of a Taylor series in s about the *sn* of the isolated cylinder. This solution provides adequate results for small separation distances and does not require root searching.

III. Results

Figure 1 shows the trajectory of the first antisymmetric mode of two identical cylinders with length to radius ratio of 200. The moment method solution using the Hallen equation agrees with those using the Pocklington form [4]. Results using the transcendental equation converge toward the moment-method curve as the separation distance is increased. The perturbation theory is in good agreement for small separations, but diverges from the correct solution when the system *sn* differs significantly from the isolated target *sn*. Figure 2 shows the trajectory of the lowest order modes associated with a system of two wires of differing length. The system *sn*'s are seen to spiral around the *sn*'s of the isolated targets and then diverge toward the origin as separation distance is increased. Since the system *sn*'s spiral near the isolated target *sn*'s for close spacing, it seems plausible that discrimination of a system of closely spaced targets might be possible using E-pulses based on the *sn*'s of the isolated targets. Preliminary results using more complicated targets within the same range cell confirm this hypothesis.

References

[1] E.J.Rothwell, et.al.,"Frequency Domain E-Pulse Synthesis and Target Discrimination," *IEEE Transactions on Antennas and Propagation*, vol. AP-35, no. 4, pp. 426-434, Apr. 1987.

[2] E.J. Rothwell and K.M. Chen, "A Hybrid E-Pulse/Least Squares Technique for Natural Resonance Extraction," *IEEE Transactions on Antennas and Propagation*, vol. AP-36, no. 3, pp. 296-298, Mar. 1988.

[3] Che-I Chuang and D.P. Nyquist, "Perturbational Formulation for Nearly Degenerate SEM Modes of Loosely-Coupled Bodies," National Radio Science Meeting, Jan. 1984.

[4] K.R.Umashankar, et.al.,"Scattering by a Thin Wire Parallel to a Ground Plane Using the Singularity Expansion Method," *IEEE Transactions on Antennas and Propagation*, vol. AP-23, no. 2, pp. 178-184, Mar. 1975.

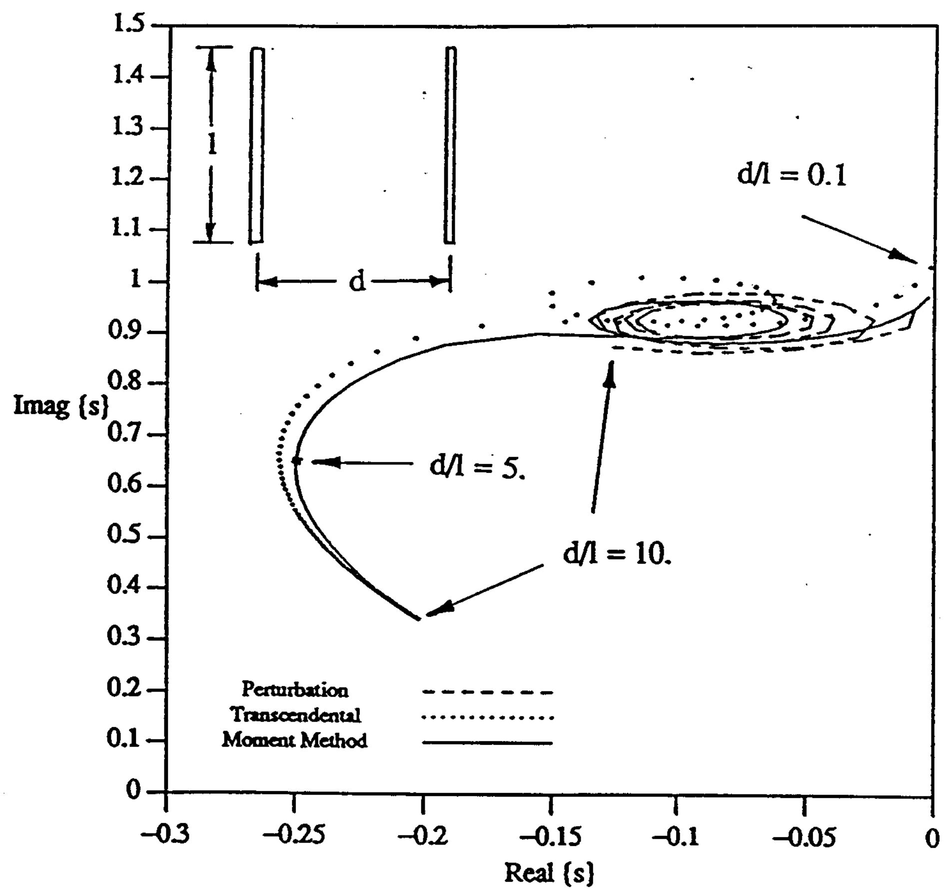

Figure 1. Trajectory of first antisymmetric mode as separation distance d is varied. Note s is normalized by πc.

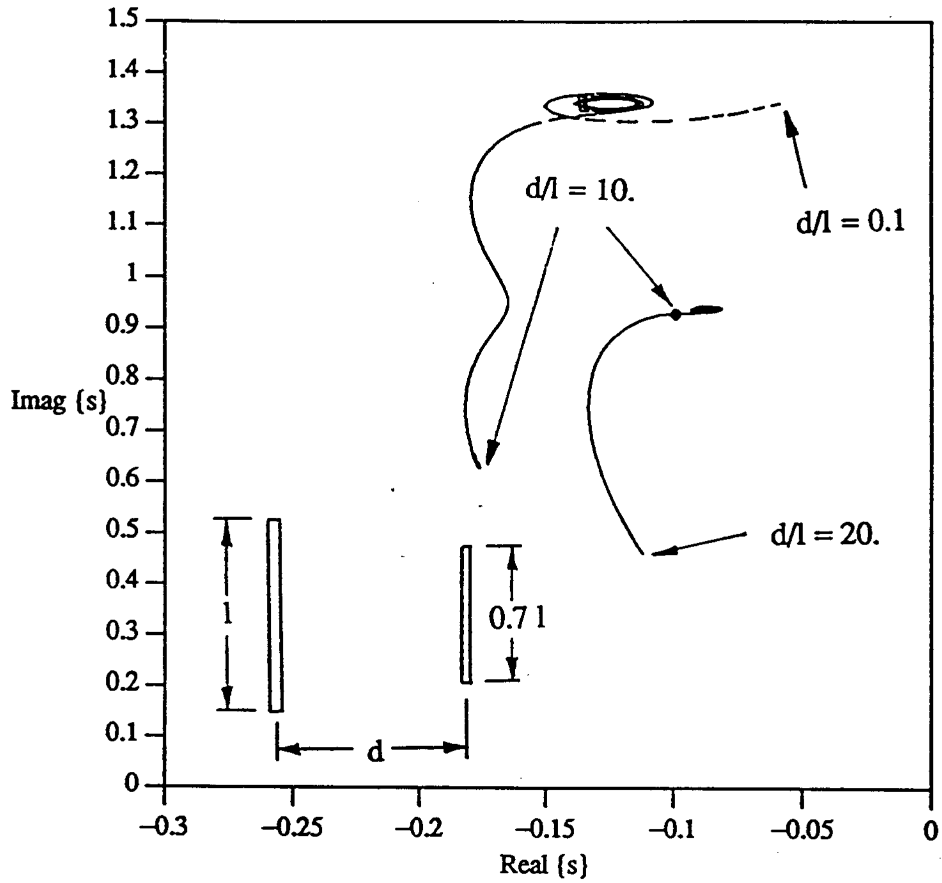

Figure 2. Trajectory of lowest order modes of two wires of different length. Length to radius ratio = 200 for each wire. Note s is normalized by πc.

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OVERVIEW

- E-Pulse Technique
- Multiple Target Scenario
- Theoretical Formulation
    - Moment Method
    - Perturbation - Transcendental
    - Perturbation - Approximate
- Results
    - Theoretical mode frequencies
    - Experimental mode frequencies
    - Discrimination Ratios

TARGET DISCRIMINATION USING E-PULSES

- Based on natural frequencies of target

- From S.E.M.

$$r(t) = \sum_{n=1}^{N} a_n e^{\sigma_n t} \cos(\omega_n t + \phi_n) \quad t > T_L$$

T_L = start of Late Time

- E-Pulse waveform

$$r(t) * e(t) = 0 \qquad t \geq T_L + T_e$$

T_e = duration of $e(t)$

- Definition of $e(t)$ implies

$$\mathcal{L}\{e(t)\} = E(s) = 0$$

for $s = s_n = \sigma_n + j\omega_n \; ; \; n = 1, 2, 3, \ldots N$

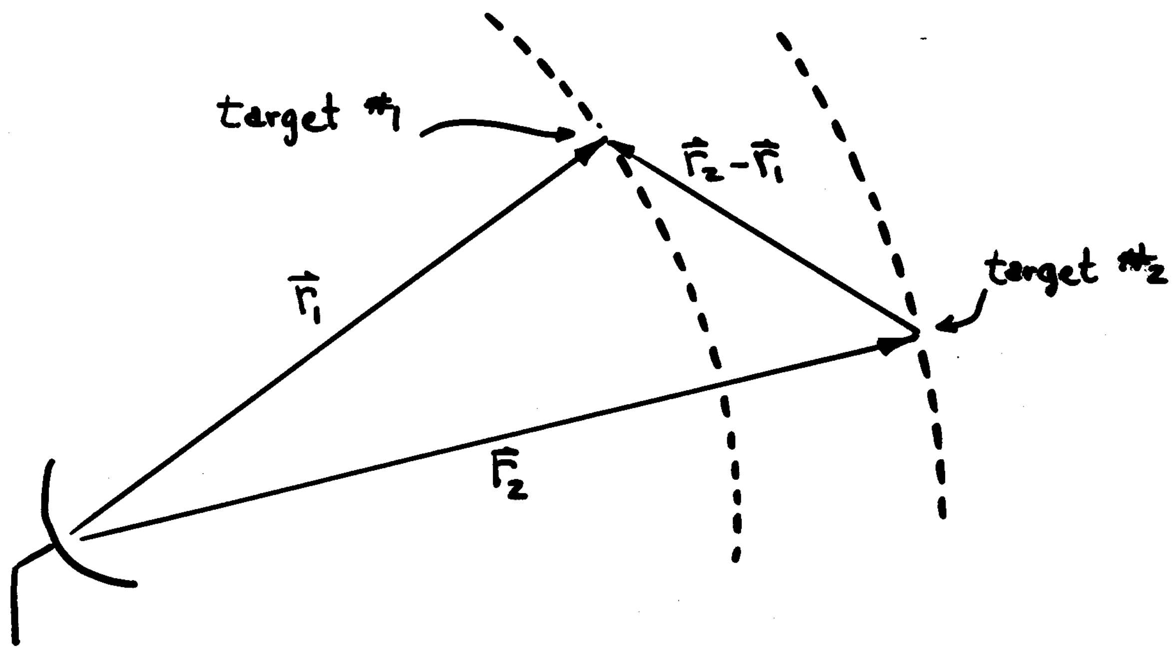
- $e(t)$ is unique

$$R_i(t) * E_j(t) \neq 0 \qquad i \neq j$$

- The A_n are measured in laboratory.

- Convolutions performed numerically in a computer

MULTIPLE TARGET SCENARIO



CASE 1. $|\vec{r}_2| > |\vec{r}_1|$, $|\vec{r}_2 - \vec{r}_1|$ large

Use time gating.

CASE 2. $|\vec{r}_1| = |\vec{r}_2|$, $|\vec{r}_2 - \vec{r}_1|$ large

Use narrow beam and/or successive convolutions.

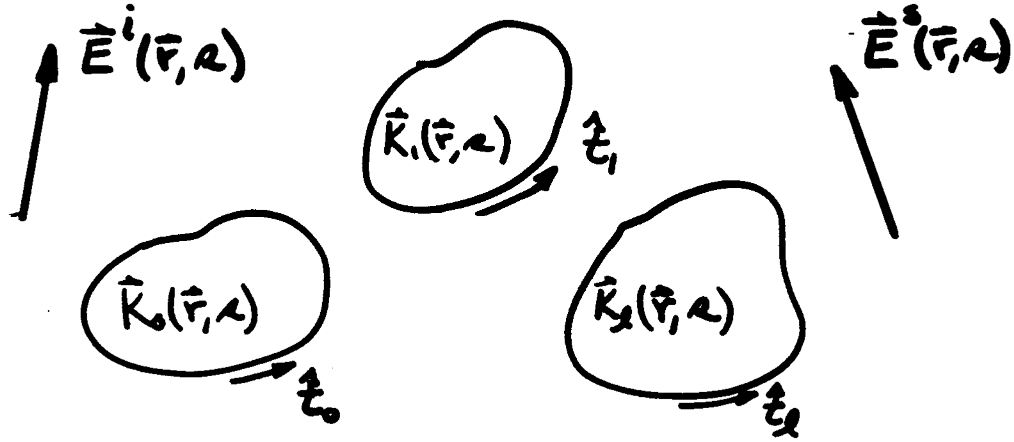
$r(t) * e_1(t) * e_2(t) = 0 \quad t > T_{L_{max}} + T_{e_1} + T_{e_2}$

★ CASE 3. $|\vec{r}_1| = |\vec{r}_2|$, $|\vec{r}_2 - \vec{r}_1|$ small

System behavior must be considered.

Is E-pulse technique useful?

THEORETICAL DETERMINATION OF NATURAL FREQUENCIES



N CONDUCTING SCATTERERS

1. $$\hat{t}_m \cdot \left[\vec{E}^i(\vec{r}, s) + \vec{E}^s(\vec{r}, s) \right] = 0$$

2. $$\vec{E}^s(\vec{r}, s) = \int_S \overleftrightarrow{G}(\vec{r}, \vec{r}', s) \cdot \vec{K}(\vec{r}, s)\, ds'$$

3. $$\overleftrightarrow{G}(\vec{r}, \vec{r}', s) = -\gamma \mu_0 \left[\frac{1}{\gamma^2} \nabla\nabla + \overleftrightarrow{I} \right] G(\vec{r}, \vec{r}', s) + (\ldots)$$

$$\gamma \triangleq \frac{s}{c} \qquad G(\vec{r}, \vec{r}', s) = \frac{e^{-\gamma R}}{4\pi R} \qquad R = |\vec{r} - \vec{r}'|$$

4. $$\hat{t}_m \cdot \sum_{\ell=1}^{N} \int_{S_\ell} \overleftrightarrow{G}(\vec{r}, \vec{r}', s) \cdot \vec{K}_\ell(\vec{r}, s)\, ds' = -\hat{t}_m \cdot \vec{E}^i(\vec{r}, s)$$

5. $$\vec{E}^i(\vec{r}, s) = 0$$ $\Longrightarrow$ **Definition of natural mode frequencies and currents.**

Substitute 3. into $\overset{4}{2}$. and invoke 5.

$$6. \quad \sum_{\ell=1}^{N} \int_{S_\ell} \left[\nabla' \cdot \vec{K}_{\ell n}(\vec{r}';\rho_n) (\hat{t}_m \cdot \nabla) - \gamma_n^2 \hat{t}_m \cdot \vec{K}_{\ell n}(\vec{r}';\rho_n) \right] \cdot G(\vec{r},\vec{r}';\rho_n) = 0$$

Specialize to array of <u>thin wires</u> parallel to z-axis

$$7. \quad \left(\frac{\partial^2}{\partial z^2} - \gamma^2 \right) \sum_{\ell=1}^{N} \int_{z_\ell^\circ - L_\ell}^{z_\ell^\circ + L_\ell} I_\ell(z') \, K_{m\ell}(z|z') \, dz' = 0 \; ; \; \gamma = \gamma_n .$$

z_ℓ° - center of ℓ^{th} wire

L_ℓ - half length of ℓ^{th} wire

$$K_{m\ell}(z|z') = \frac{e^{-\gamma R_{m\ell}(z|z')}}{R_{m\ell}(z|z')}$$

$$R_{m\ell}(z|z') = \sqrt{(z-z')^2 + d_{m\ell}^2}$$

$$d_{m\ell} = \begin{cases} \text{radius of } \ell^{th} \text{ wire} & \ell = m \\ \text{axial spacing} & \ell \neq m \end{cases}$$

MOMENT METHOD

Convert to Hallen form for monopoles over ground ($z_\ell^\circ = 0$)

8. $$\sum_{\ell=1}^{N} \int_0^{L_\ell} I_\ell(z')\, \bar{K}_{m\ell}(z|z')\, dz' = B_m \cosh \gamma z$$

$$\bar{K}_{m\ell}(z|z') = K_{m\ell}(z|z') + K_{m\ell}(z|-z')$$

Expand $I_\ell(z')$ with pulse functions. Apply point matching. Invoke B.C. @ $z = L_\ell$.

9. $$\tilde{A}(s)\, c = 0 \qquad s = s_n$$

10. $$\det \tilde{A}(s) = 0 \qquad s = s_n$$

Use numerical search for s_n. Find null$\{\tilde{A}(s_n)\}$ for mode currents.

PERTURBATION SOLUTION

Assume currents from isolated targets

11. $$\vec{K}_{\ell n}(\vec{r}', s_n) \approx a_\ell \vec{K}_\ell^\circ(\vec{r}', s_\ell^\circ)$$

12. $$\hat{t}_m \cdot \sum_{\ell=1}^{N} a_\ell \int_{S_\ell} \overleftrightarrow{G}(\vec{r}, \vec{r}', s_n) \cdot \vec{K}_\ell^\circ(\vec{r}', s_\ell^\circ)\, ds' = 0$$

Use weighting function $\int_{S_m} dS\ \vec{K}_m^\circ(\vec{r}, s_m^\circ) \cdot$

13. $$\tilde{C}(s)\, a = 0 \qquad s = s_n$$

14. $$c_{m\ell} = \int_{S_m} K_m^\circ(\vec{r}, s_m^\circ) \int_{S_\ell} \overleftrightarrow{G}(\vec{r}, \vec{r}', s_n) \cdot \vec{K}_\ell^\circ(\vec{r}', s_\ell^\circ)\, ds'\, ds$$

15. $$\det\ \tilde{C}(s) = 0 \qquad s = s_n$$

Use numerical root search to find s_n.

ADVANTAGE: FEWER MODES

APPROXIMATE GREEN'S FUNCTION

Use isolated target currents and

16. $$\overleftrightarrow{G}(\vec{r},\vec{r}\,',\ell) = \overleftrightarrow{G}(\vec{r},\vec{r}\,',\ell_\ell^\circ) + \left.\frac{\partial \overleftrightarrow{G}(\vec{r},\vec{r}\,',\ell)}{\partial \ell}\right|_{\ell=\ell_\ell^\circ} (\ell-\ell_\ell^\circ)$$

to obtain,

17. $$\hat{t}_m \sum_{\ell=1}^{N} a_\ell \int_{S_\ell} \left[\overleftrightarrow{G}(\ell_\ell^\circ) + \left.\frac{\partial \overleftrightarrow{G}(\ell)}{\partial \ell}\right|_{\ell=\ell_\ell^\circ} (\ell-\ell_\ell^\circ)\right] \cdot \vec{K}_\ell^\circ(\ell_\ell^\circ)\, ds' = 0$$

Retain leading nonvanishing terms.

For $m = \ell$, note that

18. $$\hat{t}_m \cdot a_m \int_{S_m} \overleftrightarrow{G}(\ell_m^\circ) \cdot \vec{K}_m^\circ(\vec{r}\,', \ell_m^\circ)\, ds' \triangleq 0$$

For $m \neq \ell$, assume

19. $$\left.\frac{\partial \overleftrightarrow{G}(\ell)}{\partial \ell}\right|_{\ell=\ell_\ell^\circ} \ll \overleftrightarrow{G}(\ell_\ell^\circ)$$

To obtain

20. $$a_m(\ell-\ell_m^\circ)\,\hat{t}_m \cdot \int_{S_m} \left.\frac{\partial \overleftrightarrow{G}(\ell)}{\partial \ell}\right|_{\ell=\ell_\ell^\circ} \cdot \vec{K}_m^\circ(\vec{r}\,', \ell_m^\circ)\, ds'$$

$$+ \sum_{\substack{\ell=1 \\ \ell\neq m}}^{N} a_\ell\, \hat{t}_m \int_{S_\ell} \overleftrightarrow{G}(\ell_\ell^\circ) \cdot \vec{K}_\ell^\circ(\ell_\ell^\circ)\, ds' \approx 0$$

Note also that

21. $$\vec{E}_\ell^\circ(\vec{r}) \triangleq \int_{S_\ell} \overleftrightarrow{G}(\lambda_\ell^\circ) \cdot \vec{K}_\ell^\circ(\vec{r}', \lambda_\ell^\circ)\, ds' \qquad \vec{r} \notin S_\ell$$

Now operate on L.H.S with $\int_{S_m} ds\, \vec{K}_m^\circ(\vec{r}, \lambda_m^\circ) \cdot$

to get

22. $$\tilde{C}(\lambda)\, a = 0 \implies \det \tilde{C}(\lambda) = 0 \;;\; \lambda = \lambda_n$$

where

$$c_{m\ell} = \begin{cases} (\lambda - \lambda_m^\circ)\, \overline{C}_{m\ell} & m = \ell \\ \int_{S_m} \vec{K}_m^\circ(\vec{r}, \lambda_m^\circ) \cdot \vec{E}_\ell^\circ(\vec{r})\, ds & m \neq \ell \end{cases}$$

and

$$\overline{C}_{m\ell} = \int_{S_m} \vec{K}_m^\circ(\vec{r}, \lambda_m^\circ) \cdot \int_{S_m} \left.\frac{\partial \overleftrightarrow{G}(\lambda)}{\partial \lambda}\right|_{\lambda = \lambda_m^\circ} \cdot \vec{K}_m^\circ(\vec{r}', \lambda_m^\circ)\, ds'\, ds$$

ADVANTAGE: $N = 2 \implies$ NO NUMERICAL ROOT SEARCH

Example: Two Conducting Targets

$$\det \begin{pmatrix} (\lambda-\lambda_1^\circ)\bar{C}_{11} & C_{12} \\ C_{21} & (\lambda-\lambda_2^\circ)\bar{C}_{22} \end{pmatrix} = 0$$

$$(\lambda-\lambda_1^\circ)(\lambda-\lambda_2^\circ)\bar{C}_{11}\bar{C}_{22} - C_{12}C_{21} = 0$$

$$\lambda = \frac{\lambda_1^\circ+\lambda_2^\circ}{2} \pm \sqrt{\left(\frac{\lambda_1^\circ-\lambda_2^\circ}{2}\right) + c^2}$$

with

$$c^2 \triangleq \frac{C_{12}C_{21}}{\bar{C}_{11}\bar{C}_{22}}$$

So $\quad \lambda = \bar{\lambda} \pm \delta \quad$ with $\quad \bar{\lambda} = \dfrac{\lambda_1^\circ+\lambda_2^\circ}{2}$,

$$\delta = \sqrt{\Delta^2 + c^2}\ , \quad \Delta = \frac{\lambda_1^\circ-\lambda_2^\circ}{2}$$

Identical Targets $\Rightarrow$ $\lambda_1^\circ = \lambda_2^\circ$, $\Delta = 0$, $\bar{\lambda} = \lambda_1^\circ$,
$C_{12} = C_{21}$, $\quad \delta = c = C_{12}/\bar{C}_{11}$ and

$$\lambda = \lambda_0 \pm c$$

For thin wires calculation of $C_{12}, \bar{C}_{11}$ is simplified by using sinusoidal current approximation.

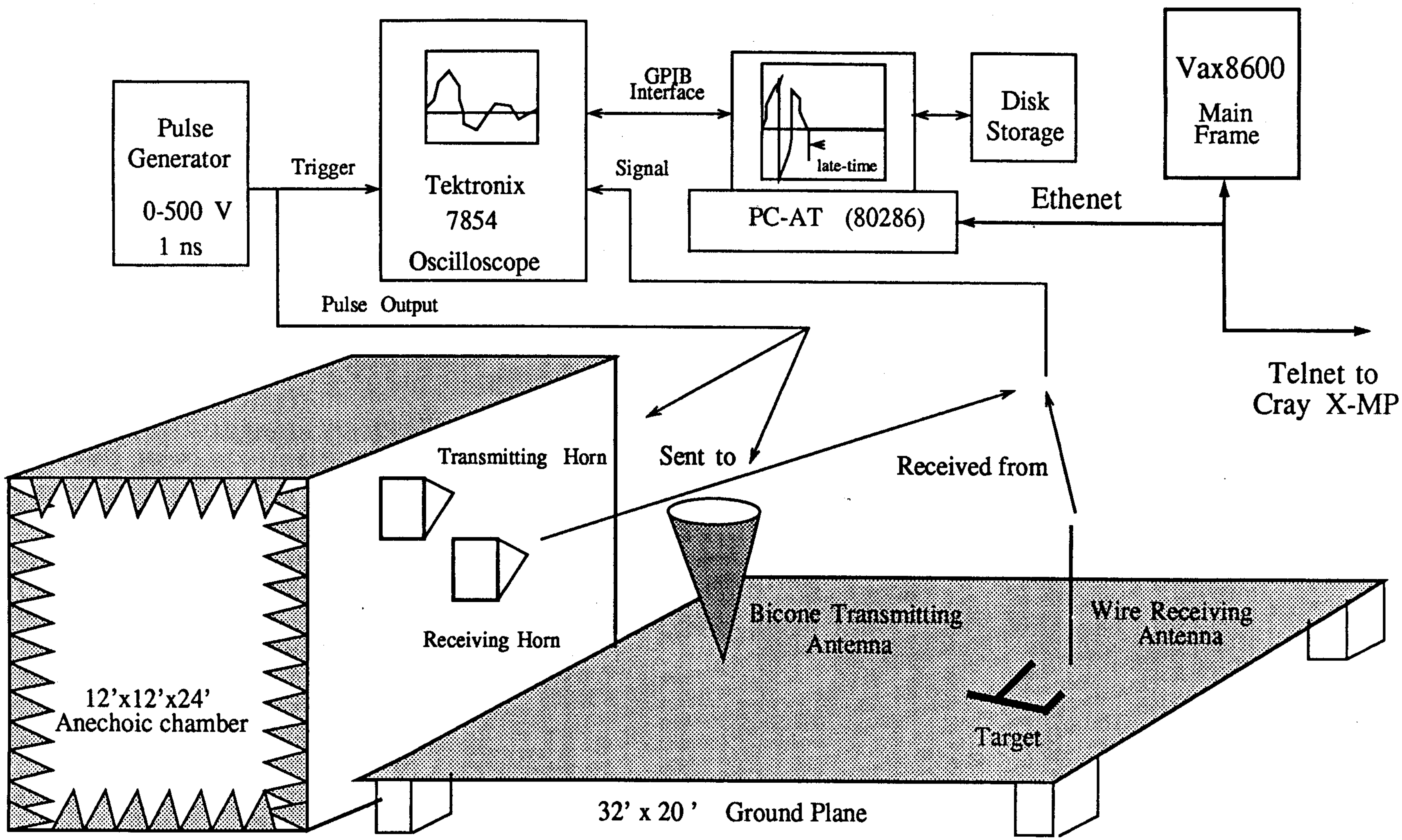
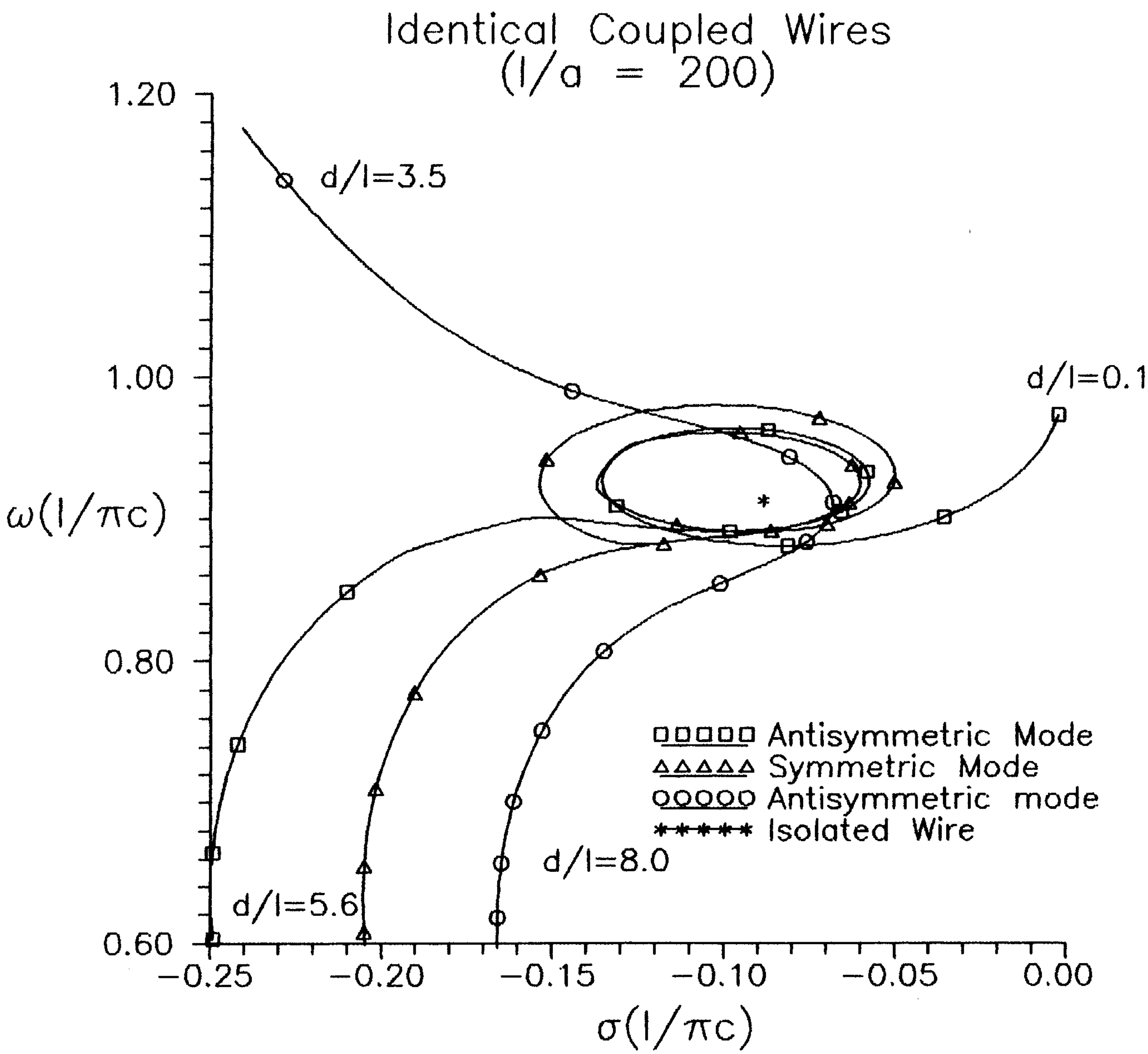
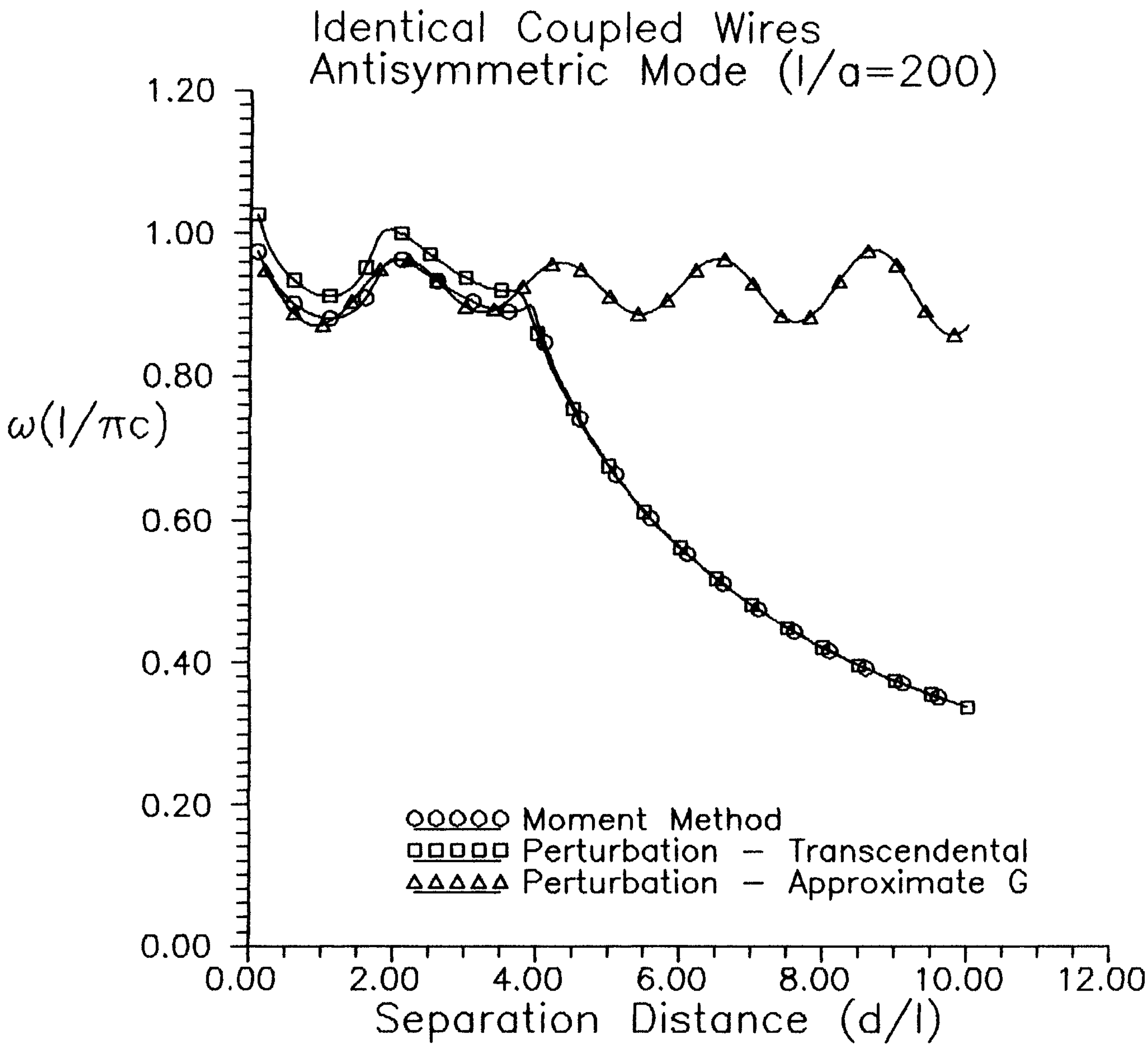
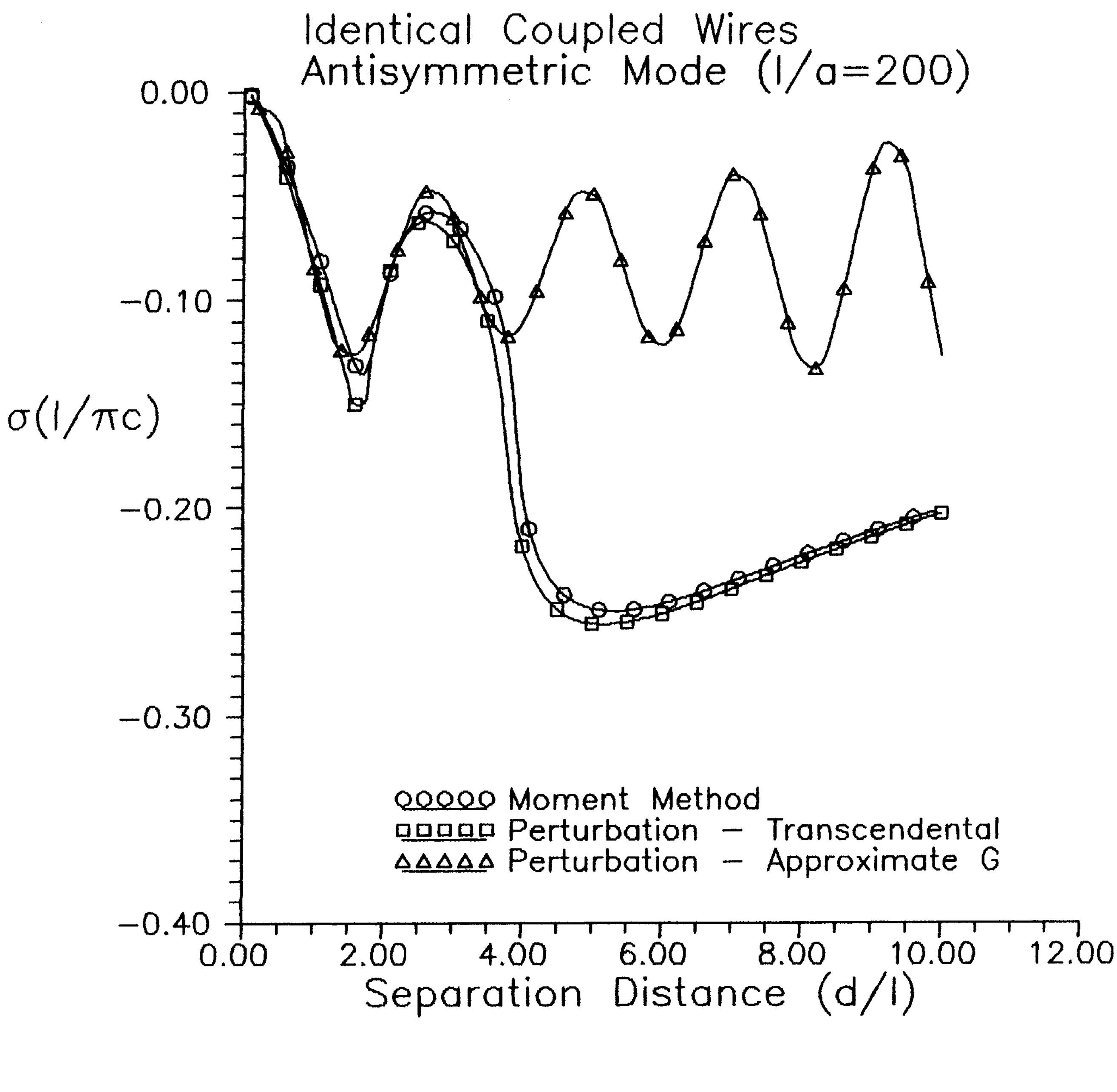


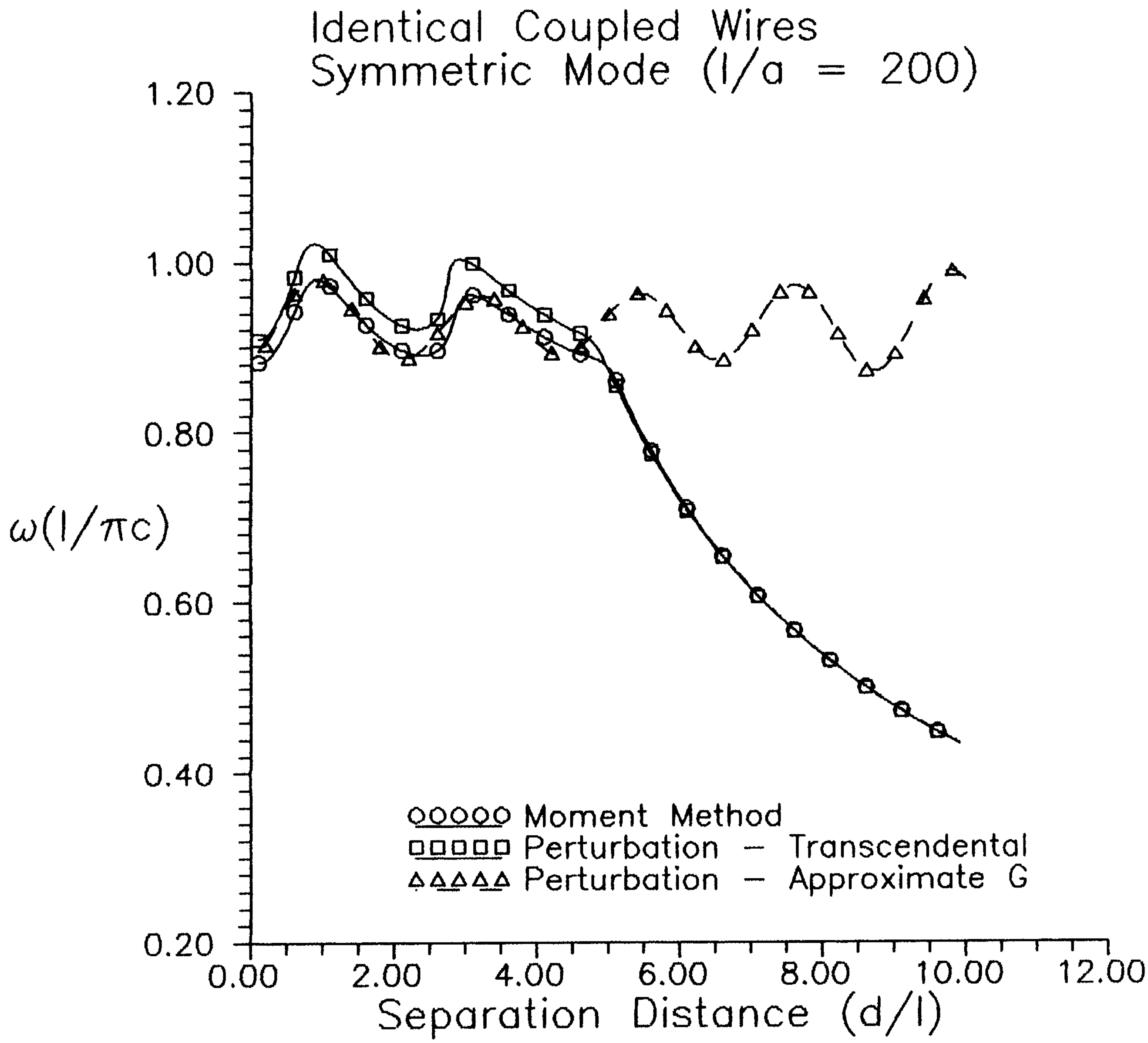
Fig. 6.1 Experiment Facilities in Electromagnetic Laboratory at Michigan State University

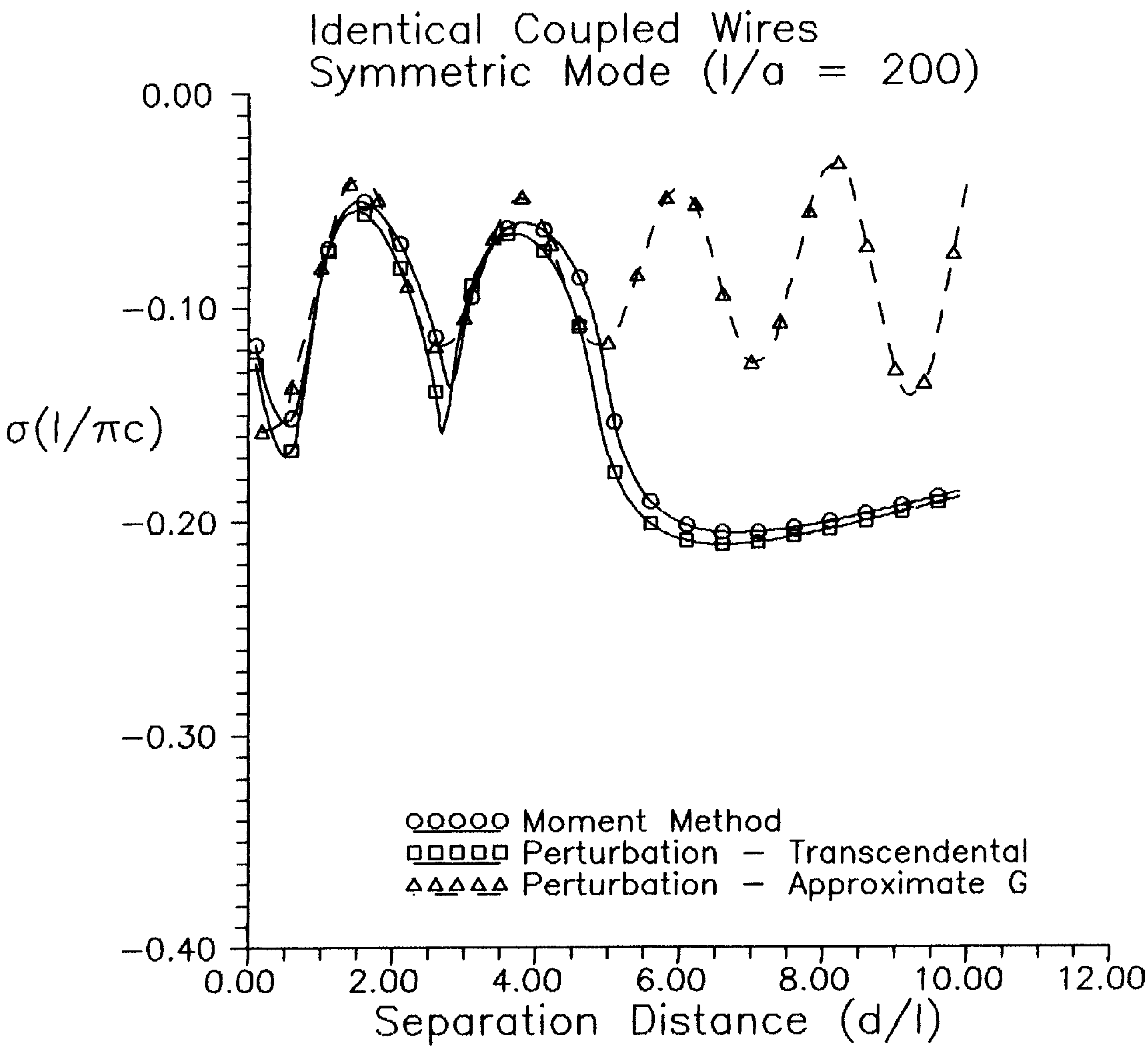

Identical Coupled Wires
(l/a = 200)
1.20
1.00
0.80
0.60
ω(l/πc)
d/l=3.5
d/l=0.1
d/l=8.0
d/l=5.6
Antisymmetric Mode
Symmetric Mode
Antisymmetric mode
Isolated Wire
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
σ(l/πc)

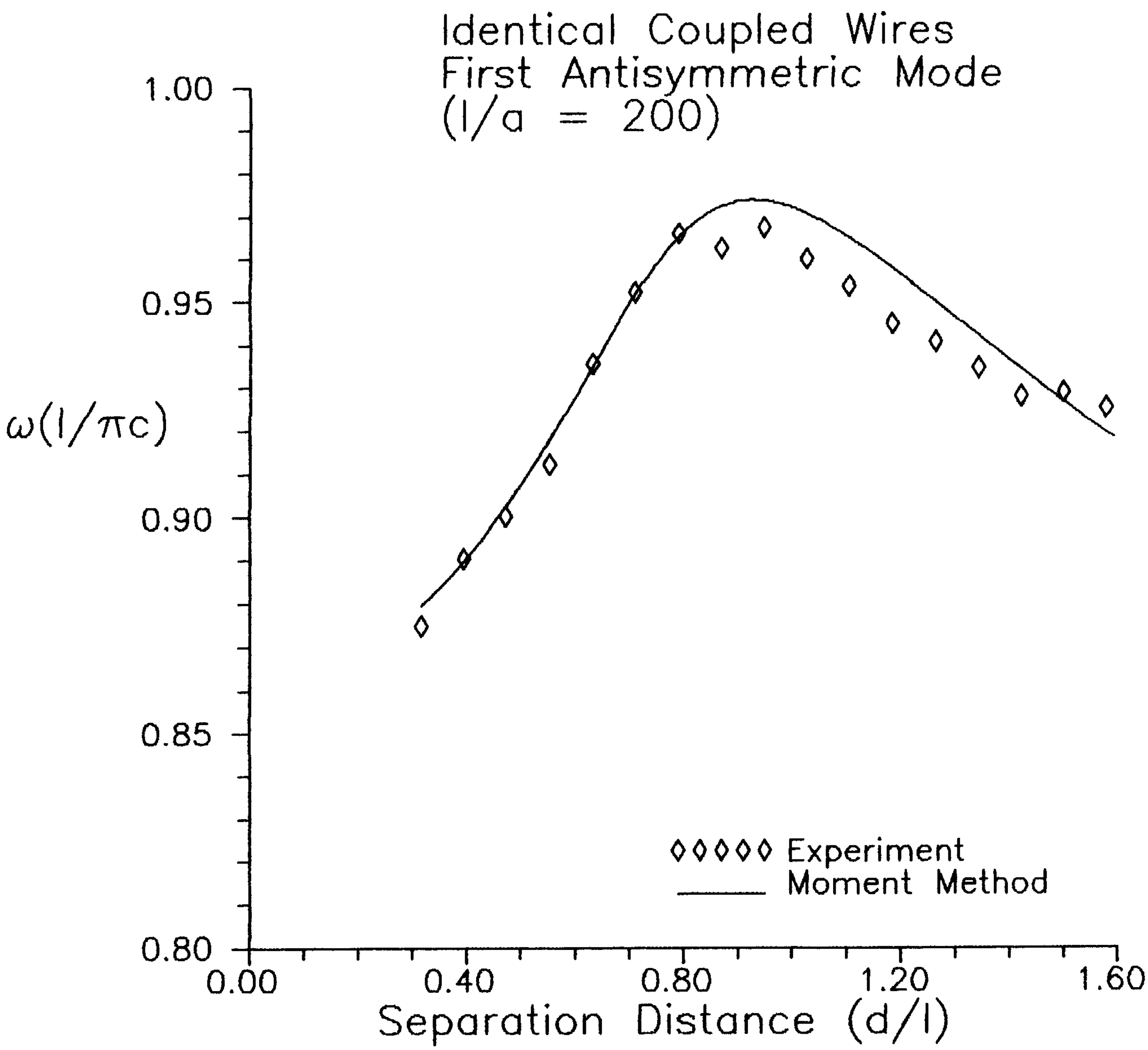


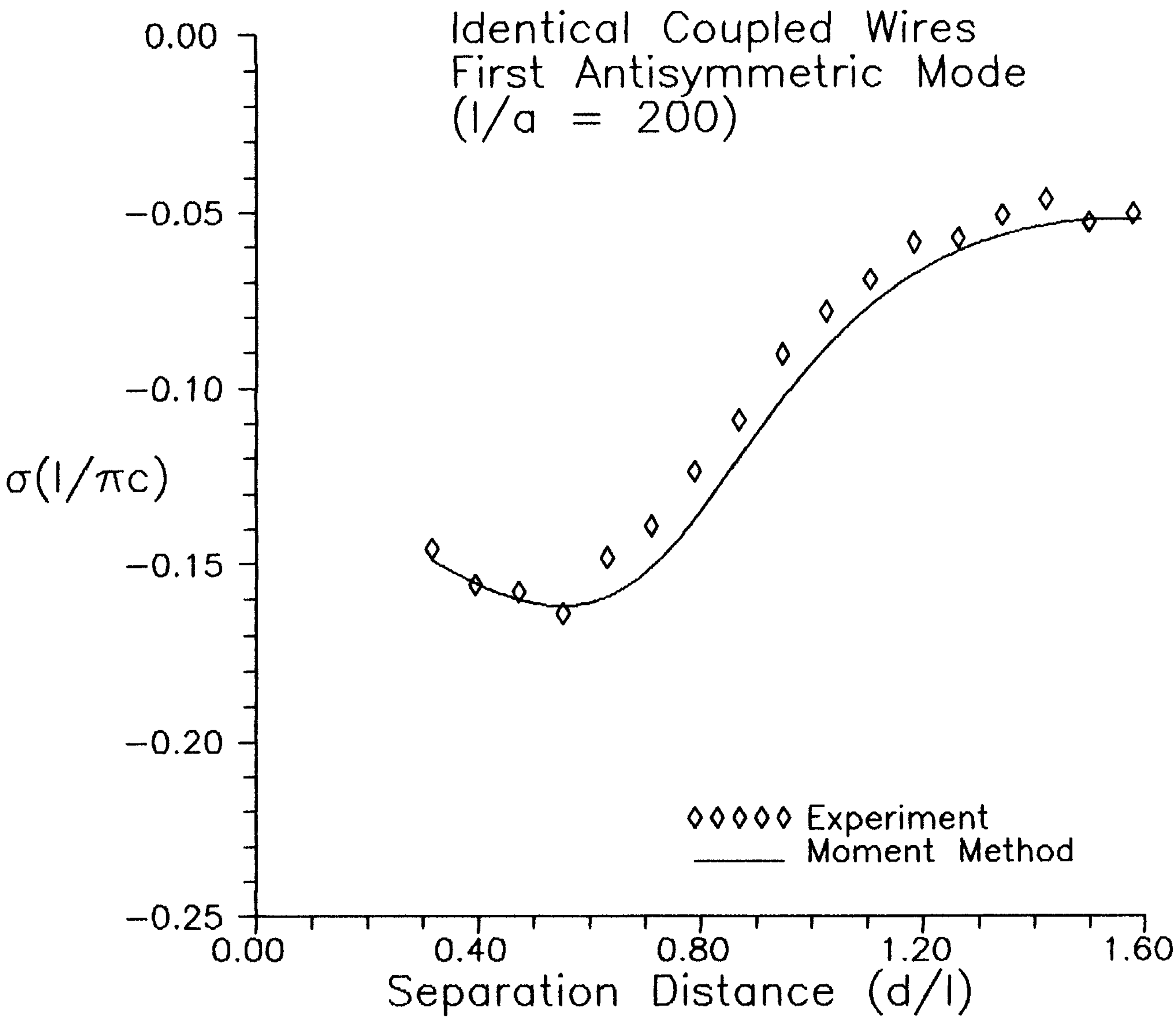
Identical Coupled Wires
Antisymmetric Mode (l/a=200)
1.20
1.00
0.80
0.60
0.40
0.20
0.00
ω(l/πc)
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Separation Distance (d/l)
Moment Method
Perturbation – Transcendental
Perturbation – Approximate G

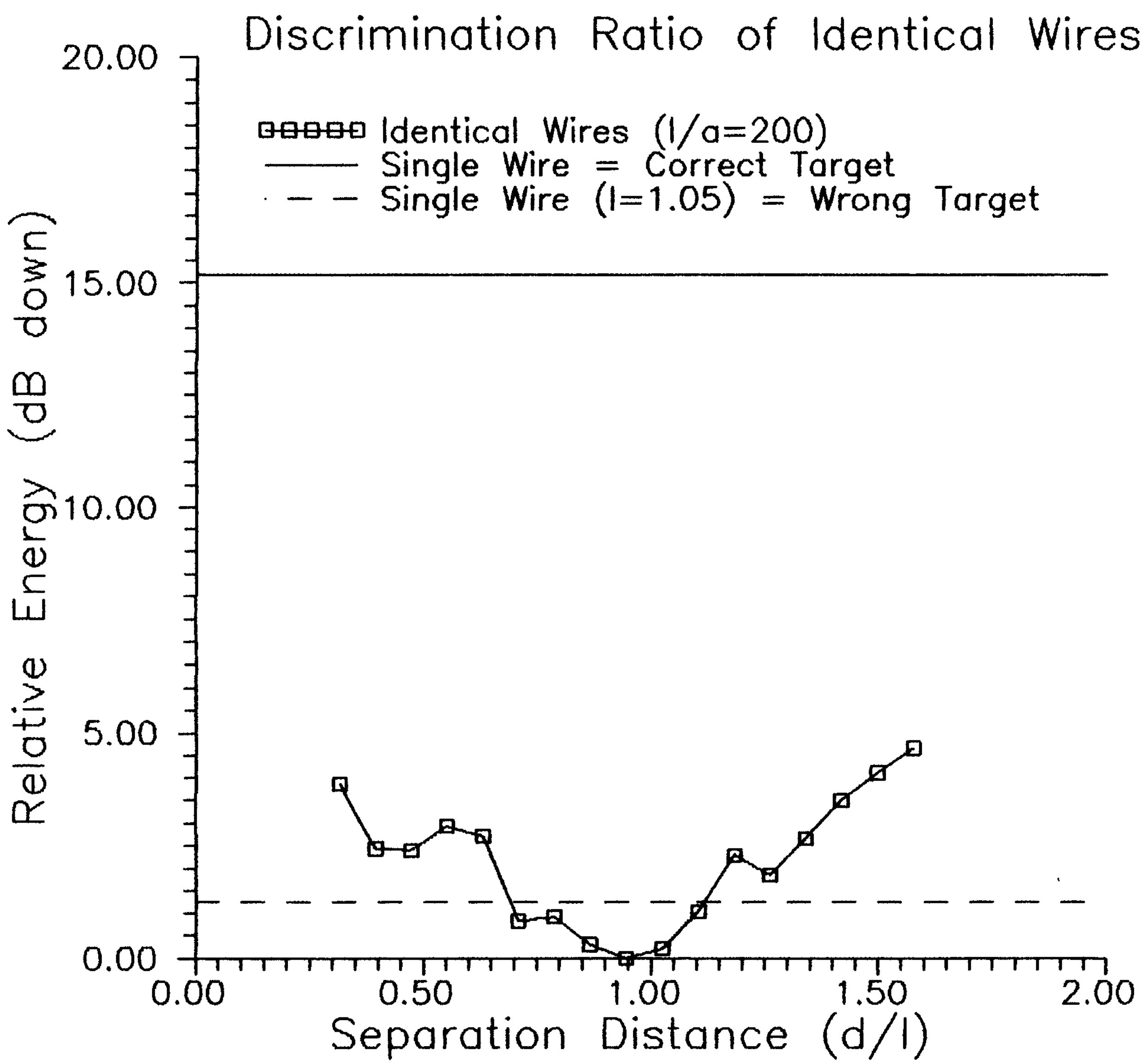

Identical Coupled Wires
Antisymmetric Mode (l/a=200)
0.00
−0.10
−0.20
−0.30
−0.40
σ(l/πc)
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Separation Distance (d/l)
Moment Method
Perturbation − Transcendental
Perturbation − Approximate G


Identical Coupled Wires
Symmetric Mode (l/a = 200)
1.20
1.00
0.80
0.60
0.40
0.20
ω(l/πc)
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Separation Distance (d/l)
Moment Method
Perturbation – Transcendental
Perturbation – Approximate G


Identical Coupled Wires
Symmetric Mode (l/a = 200)
0.00
−0.10
−0.20
−0.30
−0.40
σ(l/πc)
Moment Method
Perturbation − Transcendental
Perturbation − Approximate G
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Separation Distance (d/l)


Identical Coupled Wires
First Antisymmetric Mode
(l/a = 200)
1.00
0.95
0.90
0.85
0.80
ω(l/πc)
0.00
0.40
0.80
1.20
1.60
Separation Distance (d/l)
◊◊◊◊◊ Experiment
Moment Method


Identical Coupled Wires
First Antisymmetric Mode
(l/a = 200)
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
σ(l/πc)
0.00
0.40
0.80
1.20
1.60
Separation Distance (d/l)
◊◊◊◊◊ Experiment
Moment Method


Discrimination Ratio of Identical Wires
Identical Wires (l/a=200)
Single Wire = Correct Target
Single Wire (l=1.05) = Wrong Target
Relative Energy (dB down)
20.00
15.00
10.00
5.00
0.00
0.50
1.00
1.50
2.00
Separation Distance (d/l)