

# DETERMINATION OF RADAR TARGET SCATTERING CENTER TRANSFER FUNCTIONS FROM MEASURED DATA

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## I. Introduction

Development of successful radar target discrimination schemes using ultrawideband signatures hinges on an accurate understanding of the scattering behavior of complicated radar targets. In the time domain, a target response consists of an early-time component, which is localized and specular in nature, followed by a late-time natural mode series describing the global characteristics of the target. The temporal shape of specular responses depends on the localized geometry of the target; the sharp edge of an aircraft wing will produce a different event than the curved edge of a fuselage. This paper describes a simple technique for determining the localized transfer functions of a target from its measured time or frequency-domain response.

## II. Theory

If secondary interactions are neglected, the early-time response of a radar target to a transmitted pulse  $p(t)$  can be approximated as a series of pulse responses

$$s(t) \approx p(t) * \sum_m h_m(t) = \sum_m f_m(t) \quad (1)$$

where  $h_m(t)$  is the impulse response of the  $m^{\text{th}}$  scattering center. Altes [1] suggests that the impulse response of the  $m^{\text{th}}$  scattering center located at temporal position  $T_m$  can be expanded as

$$h_m(t) = \sum_{n=-\infty}^{\infty} a_{mn} \delta^{(n)}(t - T_m) \quad (2)$$

Here a negative value of  $n$  refers to the  $n^{\text{th}}$  integral of the delta function while a positive value of  $n$  refers to the  $n^{\text{th}}$  derivative of the delta function. This expansion is equivalent to representing the transfer function of the  $m^{\text{th}}$  scattering center as a polynomial. Using the time-shifting and differentiation theorems for Fourier transforms gives

$$H_m(\omega) = \mathcal{F}\{h_m(t)\} = e^{-j\omega T_m} \sum_n a_{mn} (j\omega)^n \quad (3)$$

Unfortunately, it is not possible to measure  $H_m(\omega)$  since only a finite portion of the spectrum can be covered in any measurement. Thus, it is necessary to deal with the band-limited pulse response of the scattering centers. Let  $F_m(\omega) = \mathcal{F}\{f_m(t)\}$  represent the band-limited transfer function of the  $m^{\text{th}}$  scattering center. Then

$$F_m(\omega) = \sum_n a_{mn} G_{mn}(\omega) \quad (4)$$

where

$$G_{mn}(\omega) = P(\omega)e^{-j\omega T_m}(j\omega)^n \quad (5)$$

and  $P(\omega)$  is the spectrum of  $p(t)$ . Thus, the pulse response of the  $m^{\text{th}}$  scattering center can be written as

$$f_m(t) = \sum_n a_{mn} g_{mn}(t) \quad (6)$$

where  $g_{mn}(t) = \mathcal{F}^{-1}\{G_{mn}(\omega)\}$ .

When the response of a target is measured in the frequency domain, the scattering-center transfer functions (4) all overlap and cannot be separated. However, if the frequency band is wide enough, the pulse responses (6) found by windowing and inverse transforming the frequency domain target response will be temporally separated. Thus, computation of the scattering center transfer functions must be done in the time domain, by calculating the unknown amplitudes  $a_{mn}$ . These then give the transfer functions through (4).

The procedure for computing the amplitudes  $a_{mn}$  uses repetitive least-squares, as follows. The measured frequency-domain scattered field response of a particular target is windowed with the function  $P(\omega)$  (to reduce unwanted truncation-induced oscillations) and inverse transformed into the time domain using the FFT. Then the function  $f_1(t)$  calculated from (6) is fit to the response with the amplitudes  $a_{mn}$  determined to minimize

$$\epsilon(T_m) = \sum_i \left[ s(t_i) - \sum_n a_{mn} g_{mn}(t_i) \right]^2 \quad (7)$$

for a certain value of  $T_m$ . The proper  $T_m$  which describes the temporal position of the scattering center with the largest energy is found by searching the entire early-time range. After the scattering center pulse response has been determined, a signal  $s_1(t)$  is formed by subtracting off  $f_1(t)$  (providing a signal with one less scattering center). Then a waveform  $f_2(t)$  is fit to  $s_1(t)$ , determining the pulse response of the scattering center with the second highest energy. This response is then subtracted off to form a signal  $s_2(t)$  and the process is repeated until all of the dominant scattering center pulse responses have been determined. To ensure reasonable accuracy it is important to calculate  $g_{mn}(t)$  carefully. For the results reported in this section, the quantity  $G_{mn}(\omega)$  is computed for a certain value of  $T_m$  and then  $g_{mn}(t)$  is found using the inverse FFT.

### III. Example

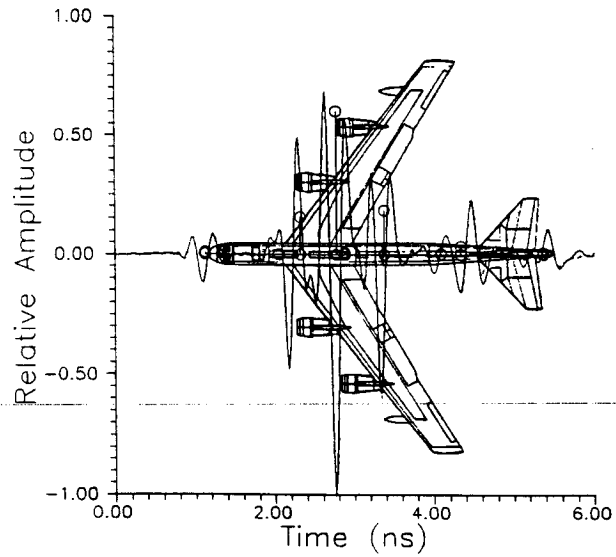
As an example, Figure 1 shows the nose-on pulse response of a 1:72 scale model B-52 aircraft measured at 601 points in the frequency band 1 to 7 GHz, windowed using a gaussian-modulated cosine function centered at 4 GHz (giving an equivalent pulse width of about 0.4 ns), and inverse transformed with a 4096 point FFT. Note that due to the relatively narrow bandwidth, the scattering center pulse responses overlap somewhat, and it is anticipated that the accuracy of the resulting scattering center transfer functions will not be optimum. Also shown in Figure 1 are the temporal positions of nine scattering centers found using the least square technique outlined above. The transfer functions of these scattering centers have been found using  $n=-2,-1,0,1,2$  in the expansion (2). The height of the circles represent the relative energy in each transfer function. Thus, the dominant specular reflection comes from the first engine mount (which coincides with the front of the second engine) and the next largest reflection comes from the second engine mount. Note that each specular reflection matches quite well with a physical feature on the target, including the trailing edges of the wing and rear wing. It is interesting to see that in terms of total energy, the reflection from the nose of the aircraft is quite small.

Once the scattering center pulse responses have been determined, the overall early-time pulse response of the target can be reconstructed using (1). This is shown in Figure 1 as the dotted line. Obviously, the reconstructed response matches extremely well, except in the latter part of the waveform where there is a small late-time component.

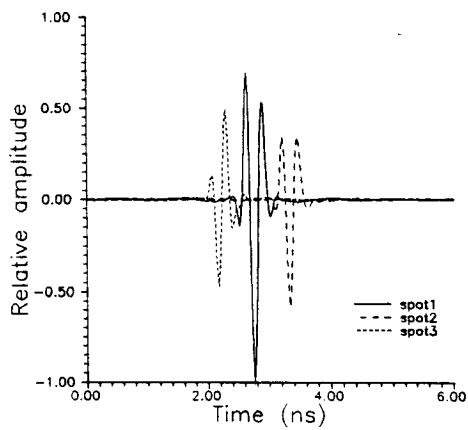
The scattering center pulse responses for the first three dominant scattering centers, calculated using (6), are shown in Figure 2. Each response has a slightly different shape, with the largest response dominated by the first integral of the equivalent pulse. This can be seen in Figure 3, which shows the scattering center transfer functions found using (4). Note that to get the true transfer function, the spectrum of the pulse  $P(\omega)$  has been divided out of (5). The first scattering center is dominated by a downward slope, indicating a  $1/\omega$  or integral response. The next two responses are relatively flat, indicating a primarily impulsive response. Similar results are seen in Figure 4 and Figure 5, which show the pulse responses and transfer functions of the next three largest scattering centers. Note that the nose response is quite close to a pure impulse over the measurement band.

### Reference

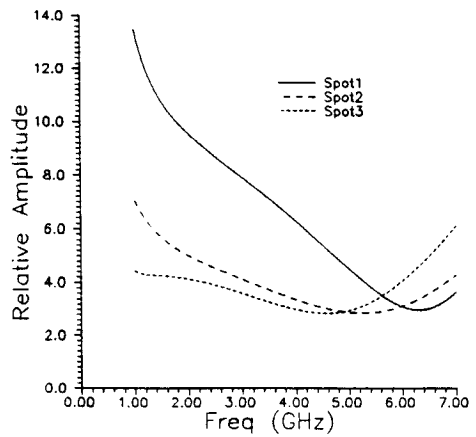
- [1] R.A. Altes, "Sonar for generalized target description and its similarity to animal echolocation systems," J. Acoust. Soc. Amer., vol. 59, pp. 97-105, January 1976.



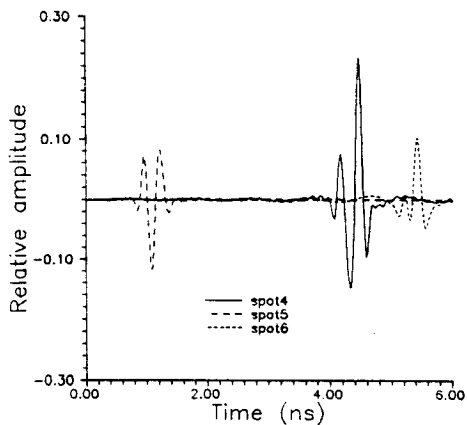
**Figure 1.** Nose-on response of B-52 aircraft.



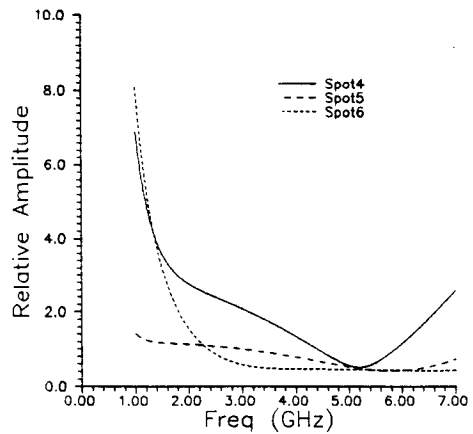
**Figure 2.** Pulse response of 1st, 2nd and 3rd brightest specular points on B-52.



**Figure 3.** Transfer function of 1st, 2nd and 3rd brightest specular points on B-52.



**Figure 4.** Pulse response of 4th, 5th and 6th brightest specular points on B-52.



**Figure 5.** Transfer functions of 4th, 5th and 6th brightest specular points on B-52.